Reply to the Comment on "Bound States in the One-dimensional Hubbard Model"

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We reply to the comment (cond-mat/9806125) by Eßler, Göhmann and Korepin, and show that their points are unfounded.

We thank the authors of the comment [1] for correcting some typos in our paper [2]. They have missed however the main point. Bound states cannot be introduced into the Lieb-Wu equations (which have been derived for real k with boundary conditions corresponding to a ring of finite length L) simply by inserting k-Λ-strings. Bound states correspond to *poles* of the S-matrix, a fact that guarantees that the wave-functions with complex k do not blow up in the $L \to \infty$ limit. Our approach provides a correct and general procedure for incorporating complex momenta. Furthermore, there is no need to appeal to a "string hypothesis".

We proceed to respond in turn to the points raised in the comment.

(I) Our Bethe equations (eq.(20)-(22) in [2]) are not new but "coincide" with Takahashi's equations (eq.(2.11a-c) in [4]) for finite system size L.

This is incorrect: The equations are different for L *finite*. Takahashi has dropped terms of order $e^{-\frac{|u|}{4}L}$ in his equations (2.11a-c). He started from the Lieb-Wu equations [3] (eq.(2,3) in [2]) for a finite system with periodic boundary conditions (PBC) and used the "string hypothesis" for both the k-Λ-strings and the Λ-strings. Keeping the neglected terms explicitly one arrives first at eq.(5)-(8) in our paper (these are simplified as they contain only 1-complexes and no Λ-strings). Eliminating from the set (5)-(8) the variables φ_l and dropping the ε-terms leads to (2.11a-c) in [4]. This procedure is incorrect because it assumes the consistency of (5)-(8) for finite L. Our approach is different: We do not proceed from (2,3) but derive the analogue to the Lieb-Wu equations (the BAC) for composite boundary conditions (CBC), which allow us to include the bound states ab initio. For an m-complex these boundary conditions read

$$F(\{x_j\}, x_1^a, \ldots, x_{2m}^a) = F(\{x_j\}, x_1^a + L, \ldots, x_{2m}^a + L)$$

where $x_1^a \ldots x_{2m}^a$ denote the positions of the particles forming the m-complex, and $\{x_j\}$ the positions of the $N-2m$ others. These boundary conditions coincide with PBC for unbound particles. Our equations (20-22) (BAC equations) are *exact* for finite L and CBC. Moreover, the reason why they do not contain the Λ-strings (which the authors of [1] suspect to be "hidden" in our notation) is that these (spin-) strings follow from a hypothesis, which is known not to be always true [5,6].

(II) The authors repeat a calculation first performed in [7] to show that in case of a single bound state excitation the relation $2\Lambda' = \sin k_1^h + \sin k_2^h$ is satisfied at half-filling. This statement has no physical meaning as is explained in column 2 on page 4 in [2]: The bound state excitation is an independent excitation only away from half-filling.

(III) The authors believe that we wanted to keep eq.(5),(6) and (8), while objecting to (7). This guess is false, as (5) and (8) contain the spin-rapidities Λ_l, whose presence causes the overdetermination of the set (5)-(8). Their derivation of the equivalence of the set (5)-(8) with the set (5),(6),(8),(B8) in appendix B of [1] does not prove the consistency of one of these sets, which follow from the k-Λ-string hypothesis - the fact that (B8) is always true for translational invariant systems with overall L-periodicity is obviously insufficient. In any case, point (III) in [1] has nothing to do with our argument for the redundancy of (7) in the infinite volume limit, as this argument does not relate to periodicity but to the equations (3) determining the conditions on the eigenvectors of the transfermatrix.

(IV) It is interesting to note that the reasoning of [1] (if correct) would lead to the following alternatives:

1) Either the Hilbert spaces for PBC respectively CBC have the same dimension as we claim and point (IV) is empty, or

2) the dimension for CBC is larger, which entails an error in [8], because there the number of states with CBC is counted (remember that the BAC equations are not based on a hypothesis), which is, according to [8], 4^L.

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